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Physical measurements in General Relativity: a new effect

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Abstract. The theory of measurements in general relativity is outlined recalling the steps which are essential to single out a physical measurement and characterize a physical effect; these steps fix the *measurement protocol* . A new general relativistic effect is illustrated as a strong case of the protocol application. In the Schwarzchild space-time, particles moving on spatially circular non-geodesic orbits with radii less than $3M$ increase their binding energy if one increases the modulus of their angular velocity hence they require a larger acceleration pointing outwards to remain on circular orbits at the same radius (Abramowicz and Lasota, 1974). More recently it was found by the Author (de Felice, 1994) that in the Kerr metric this same relativistic effect also occurs at arbitrary large distances from the rotating source, but only for counter-rotating non-geodesic spatially circular orbits and in a small range of the permitted angular velocity of revolution.

1. Introduction

A physical measurement is meaningful only if one identifies in a non ambiguous way who is the observer and what is being observed. The same observable can be the target of more than one observer so one needs a suitable algorithm to compare their measurements. This is the task of the theory of measurement in the framework of general relativity (de Felice F. and Bini D. 2010). The main product of the above theory is the formulation of a *measurement protocol*, namely a number of steps, logically consequential, which allow one to end up with a physically significant measurement. The basic protocol steps are the following, quoting de Felice F. and Bini D. (2010):

1. Identify the covariant equations which describe the phenomenon under investigation.
2. Identify the observer who makes the measurements.
3. Chose a frame adapted to that observer allowing the space-time splitting into the observer's *space* and *time*.
4. Decide whether the intended measurement is local or non-local with respect to the background curvature.
5. Identify the frame components of those quantities which are the observational targets.
6. Find a physical interpretation of the above components following a suitable criterion such as, for example, comparing with what is known in special relativity or in non-relativistic theories.

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7. Verify the degree of the residual ambiguity in the interpretation of the measurements and decide the strategy to eliminate it.

As stated, each step of the sequence requires the fulfillment of the previous ones so only at the end of the above procedure one can qualify a quantity as the result of a physical measurement. Satisfying the protocol requirements, however, does not guarantee a correct or just meaningful interpretation of the measurement. An example of a measurement which leads to the assessment of a new general relativistic effect is illustrated below.

2. The general relativistic effect

In Schwarzschild space-time, spatially circular orbits with radii $r < 3M$, M being the mass of the metric source in geometrized units, have the property that to an increase of the modulus of the angular velocity of revolution corresponds an increase of the thrust in the outward direction in order to keep the orbit circular and at the same radius. This effect, first found by Abramowicz and collaborators (Abramowicz M.A. and Lasota J.P.Bonifacio 1994), is highly counter-intuitive and has no Newtonian analogue. A reasonable interpretation is that, sufficiently close to a compact object as a black hole, an increase of the angular velocity of revolution of the orbiting particle contributes more to its weight (increasing its binding energy) than it does to the centrifugal potential (de Felice F. 1991) Although consistent with the principle of relativity, the occurrence of this effect appeared to be confined to space-time regions with high curvature and therefore not easily testable. If the space-time source, however, is a rotating one so that gravitational dragging cannot be neglected, then the above effect was found by the Author (de Felice F. 1994) to hold arbitrary away from the source and therefore testable in a gravitational field as weak as that of the Earth.

We now consider Kerr space-time; its metric reads:

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right)dt^2 - \frac{4Mar\sin^2\theta}{\Sigma}dtd\phi + \frac{A}{\Sigma}\sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 \quad (1)$$

where M is the mass of the metric source, a its specific angular momentum[1] and the functions Δ , Σ and A are given by:

$$\Delta = r^2 - 2Mr + a^2 \quad (2)$$
$$\Sigma = r^2 + a^2\cos^2\theta \quad (3)$$
$$A = (r^2 + a^2)^2 - a^2\Delta\sin^2\theta. \quad (4)$$

Its asymptotic limit leads to the Lense-Thirring solution which describes the space-time of a weakly rotating spherical source. The specific thrust associated to a non-geodesic equatorial spatially circular orbit in the Kerr metric, is given, from de Felice F. (1991) and de Felice F. and Usseglio-Tomasset S. (1991), by:

$$\tilde{f}(y, a; r) = \frac{\Delta^{1/2}}{r^2}\frac{(y - y_{g_+})(y - y_{g_-})}{(y - y_{c+})(y - y_{c-})}$$
$$\theta = \pi/2 \quad (5)$$

where

$$y_{c_\pm} = \frac{a \pm \Delta^{1/2}}{r^2} \quad (6)$$
$$y_{g_\pm} = \pm\sqrt{\frac{M}{r^3}} \quad (7)$$
$$y \equiv \frac{\Omega}{1 - a\Omega}. \quad (8)$$

Here Ω is the angular frequency of the orbital revolution as it would be measured at infinity, $y = y_{g_\pm}(r)$ is the condition for a geodesic orbit and the functions $y = y_{c_\pm}(a, r)$ describe the boundary of the permitted angular velocity pattern (de Felice F. and Usseglio-Tomasset S. 1991). Clearly, at a fixed radius r, $\tilde{f}$ diverges negatively at $y = y_{c\pm}$, it vanishes at $y = y_{g_\pm}$ and has a maximum value at $y = y_{0_\pm}$ given by

$$y_{0_\pm} = \frac{-1}{2a}\left[1 - \frac{3M}{r} \mp \sqrt{\left(1 - \frac{3M}{r}\right)^2 - \frac{4Ma^2}{r^3}}\right].$$

[1] I use geometrized units, i.e. $c = 1 = G$, c and G being respectively the velocity of light in the vacuum and the gravitational constant.

It is easy to see that y_{0_+} is always negative and the curve $y = y_{0_+}(a, r)$ extends to infinity where it goes $\sim r^{-3}$; it is larger then y_{g_-} which goes $\sim r^{-3/2}$ to the same limit; it vanishes identically when $a = 0$. Setting $y = y_{0_+}(a, r)$, the function $\tilde{f}$ takes the extreme value:

$$\tilde{f}_{ex} = \frac{\Delta^{1/2}}{r^2}\left[\sqrt{1-\frac{4Ma^2}{r^3}\left(1-\frac{3M}{r}\right)^{-2}}-1\right]\times \left[\sqrt{1-\frac{4Ma^2}{r^3}\left(1-\frac{3M}{r}\right)^{-2}}- \frac{2a^2}{r^2}\left(1-\frac{3M}{r}\right)^{-1}-1\right]^{-1}. \quad (9)$$

In the non rotating case ($a = 0$) the thrust has a maximum at $y = 0$ at all $r > 3M$, while, when the metric source is rotating ($a \neq 0$), the thrust has a maximum at $y_{0_+}(a, r)$ therefore it has an anomalous behavior in the small interval $(y_{0_+}, 0)$. Here, in fact, an increase of $|y|$ from 0 to $|y_{0_+}|$, implies an increase of the thrust outwards (being $\tilde{f} > 0$) contrary to what one expects in Newtonian mechanics, [6]. In our case the behavior of the thrust is somehow triggered by the gravitational drag remaining *hidden* when $a = 0$ until the gravitational pull becomes strong enough; as mentioned in the introduction, this occurs in the Schwarzschild space-time nearby the horizon (at $r = 2M$) where no rotational drag exists at all.

3. Conclusions

Under special conditions and contrary to what is expected in Newtonian mechanics, an increase of the angular velocity of revolution of a test particle on a spatially circular orbit, causes an increase of its apparent weight so, in order to keep the particle on the same orbit, the thrust needs to be increased in the outward direction. This is termed the relativistic thrust anomaly. This effect is contributed by both the gravitational pull and the gravitational drag; while the former alone causes the anomaly to manifest itself very close to the gravitational source hence hard to be tested, the combination of gravitational pull and drag leads this effect to manifest itself on counter-revolving orbits all the way to asymptotic distances and therefore testable with advanced technology. A way how to measure the effect in the gravitational field of the Earth was discussed in de Felice F. (1995). While its occurrence very close to the event-horizon in the non-rotating case can be reasonably understood admitting that an increase of the angular velocity contributes to the particle's weight more than it does to the centrifugal potential, its occurrence at large distances from a rotating source as it could be the Earth itself, is harder to understand in the above terms. Since the condition of maximum thrust is also that of zero precession for orbiting gyroscopes, a sensible explanation could perhaps be found in the effect of gravitational dragging on the moving frames.

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